2 The life-cycle human capital model

2.1 Introduction

The life-cycle model concerns individual investment decisions. It forms the core of human capital theory which deals with the acquisition of earnings power so crucial to understanding earnings differences. In this chapter we concentrate on two earnings patterns. The first pattern is that earnings rise with age, but at a diminishing rate so that younger workers' wages rise more quickly than older workers' wages. The second pattern is that earnings rise with education so that college graduates earn more than high school graduates, and in turn high school graduates earn more than primary school graduates. These observations are prevalent in data not only for the US and England, but also for all countries for which earnings data are available. The two patterns are universal and hence worthy of an explanation.

The chapter sets the stage by presenting statistical evidence. It then explores the life-cycle human capital model graphically, and ends by depicting the model algebraically.

2.2 The age–earnings profile

Earnings generally rise with age at a decreasing rate. This can be illustrated graphically in what is called an age–earnings profile as shown in figure 2.1. Typically age or labourforce experience is measured along the horizontal axis, and earnings along the vertical axis. The relationship between the two is depicted by the concave graph. Earnings rise quickly at young ages, but growth tapers off so that a peak is reached at about age fifty-five, and then earnings decline.

To make such a graph one ideally should have cohort data. Cohort data are information for a given generation of individuals each year of their life. Accumulating such data entails following the individuals over a forty-five year time period, for example, and hence is costly and time consuming. Asking

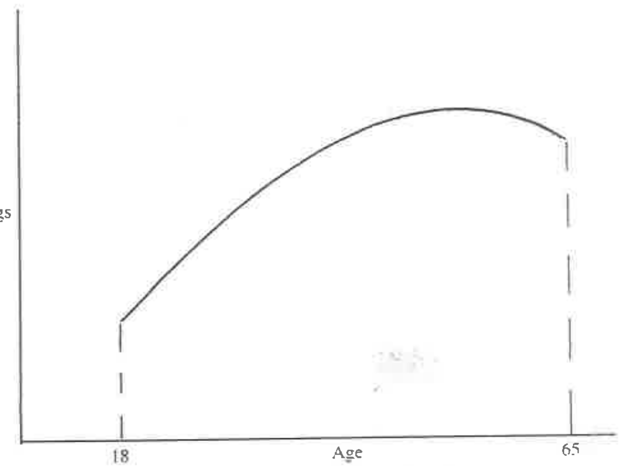

Figure 2.1 An age–earnings profile

individuals retrospectively historical questions about prior earnings often leads to erroneous data caused by problems of recalling the past. Further, even if one were to successfully conduct such a cohort earnings survey, the data would necessarily be incomplete since the young would not have had the opportunity to have worked long enough to yield a complete work history. Even the University of Michigan's Panel Study of Income Dynamics, which follows individuals over a twenty year period (since 1968), does not have enough datapoints to give complete work histories. Therefore earnings profiles are most often constructed using cross-sectional data.

Cross-sectional data are data collected from a random population at a point in time. Such data contain information for various population groups: whites, blacks, males, females, Hispanics, urban, rural, highly educated, etc. We will concentrate in this chapter on white males, and deal with other groups in later chapters.

The profile itself is computed by taking average earnings at each age for a particular demographic group – though researchers concerned with obtaining 'smooth' profiles often compute 'moving averages' of the wage rates. When using cross-sectional data it is usually assumed that the cross-section is equivalent to following an individual over his or her lifetime. This means that all individuals within the particular demographic group under study should be considered identical so that the earnings profile can be taken as depicting the change in earnings as a given individual moves through life.

Figure 2.2 depicts a set of age–earnings profiles for eight schooling groups. These profiles are not smoothed by moving average techniques and thus

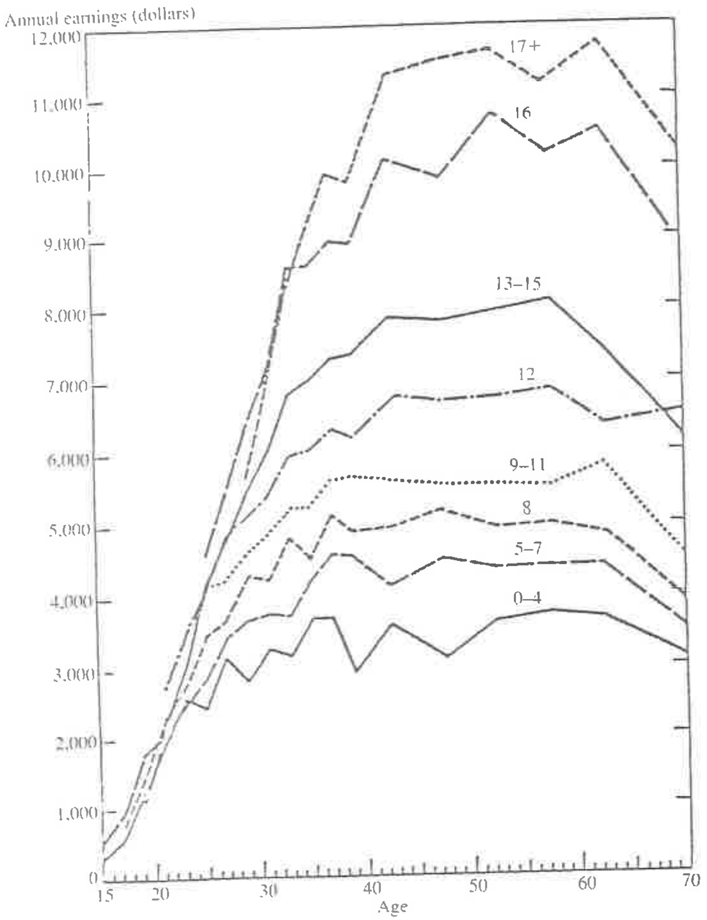

Figure 2.2 Age–earnings profiles of white non-farm men by schooling level, 1959. *Source:* Mincer, 1974, 66

oscillate a bit more than the smooth hypothetical age–earnings profile depicted in figure 2.1.

The bunching of profiles in the lower left of the figure arises because those with higher levels of schooling begin work at later chronological ages. Those with low schooling but more work experience often find their wages have risen to the entry wage of the better educated. Partly for this reason one gets a clearer picture of life-cycle wages by concentrating on experience–earnings profiles. Figure 2.3 is such a diagram. Rather than age, experience is on the horizontal axis.

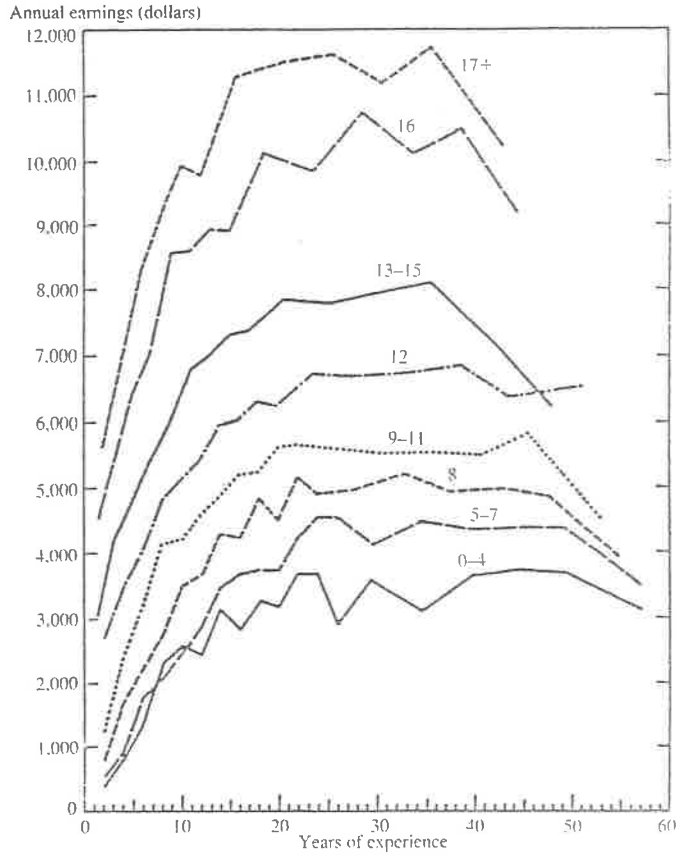

Figure 2.3 Experience–earnings profiles for white non-farm men by schooling level, 1959. *Source:* Mincer, 1974, 67

In figure 2.3 the two earnings patterns alluded to earlier are eminently clear: (1) earnings rise with age at a diminishing rate and (2) earnings profiles are higher the higher one's level of schooling. In the remainder of this chapter, we shall develop the life-cycle human capital model to find out why these patterns emerge.

2.3 The human capital model

Human capital theory explains earnings in terms of job skills acquired in school and on the job. The basic point is that a current earnings sacrifice or cost is incurred in order for a future benefit. This is the definition of an investment.

Were a current cost incurred for a current benefit, the expenditure would fall in the category of consumption. Investment in people is termed investment in human capital by analogy with investment in machinery, which is called physical capital.

Just as there is difficulty in measuring physical capital, since the value of a machine depends on the discounted future profits from the machine, not on some historic cost ('book value') of the machine, so there are questions concerning how to measure human capital. Here we deal with human capital abstractly, and assume a measure of human capital, namely 'eds', just as economists have invented 'utils' to conceptualise utility. Eds represent skill units – degrees, qualifications, on-the-job experience – acquired by individuals throughout life. One's stock of human capital at any age is related to the number of eds purchased at each age, so that the stock of human capital is the sum of human capital purchased in all prior years (minus depreciation which we shall discuss later). An individual's earnings are proportional to his or her human capital stock, the factor of proportionality being the wage or 'human capital rental rate' per ed. The greater one's human capital ed accumulation, the higher one's earnings.

If the human.capital model is to be applicable to the two questions raised in this chapter, we need to demonstrate that 'eds' are purchased at different rates over the life cycle. For earnings to rise more quickly at younger ages, human capital purchases would necessarily have to take place in greater amounts for the young relative to the old. And for earnings increase to slow down as a person ages, human capital purchases have also to taper off as one ages. Similarly human capital acquisition needs to be positively related to schooling. In order to assess these propositions concerning human capital acquisition we need to establish motives for buying human capital, so that the problem can be studied within a framework of rational choice.

The dichotomous investment decision

Imagine two hypothetical age–earnings profiles such as those shown in figure 2.4, depicting choices for an eighteen year old individual, say. The figure shows two possible choices: to go to work or to go to school (college) for at least another year. If one works it is possible to earn \$G initially and then have earnings rise to the profile *Y*(high school). On the other hand if one obtains an extra year of college, one would have zero earnings in the first year, and incur direct costs of tuition, books, etc., thus ending up with negative earnings, −\$C, between age eighteen and nineteen. However, after age nineteen, earnings profile *Y*(college) would ensue. Going to school enhances future earnings by the vertical distance between OA and OB, yet has a direct cost depicted by the two areas comprised of the direct outlay

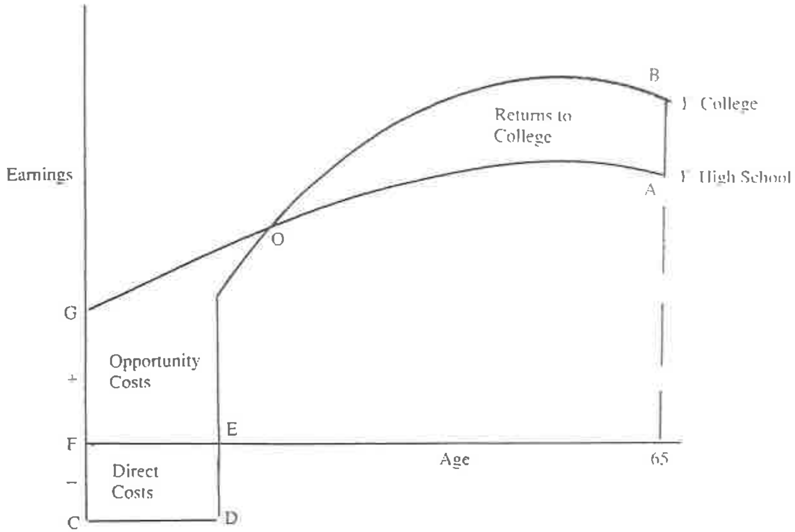

Figure 2.4 Earnings profiles for high-school and college

on books, tuition, etc. (area DEFC), and indirect opportunity costs (area GFEO).

Deciding whether to invest requires comparison of the 'present value' of future benefits with direct costs. Benefits consist of the difference between the two earnings profiles, ΔY, from age nineteen to retirement. The present value of this stream of benefits discounted at rate i, PV(i), is approximately (see appendix):

$$PV(i) \cong \Delta Y / i.$$

Investment should occur as long as costs, C, are less than the present value of benefits:

$$C < \Delta Y / i.$$

For example if $\Delta Y = \$2{,}000$ and $i = 10\%$, the cost of going to college would have to be less than \$20,000 (= 2,000/0.1).

The investment criterion is more usually stated in terms of the 'internal rate of return' (IR) – that discount rate that equates cost and present value of returns (see appendix):

$$PV(IR) = \Delta Y / IR = C.$$

On this criterion, if IR is greater than the market rate of interest plus a risk premium, then the investment is worth buying. A 'demand for education' curve can be drawn up, relating *IR* to years of investment – the first years having a high *IR* (because foregone earnings costs are low, while the benefits of learning to read and write are large), and later years having a lower *IR*. Education is bought so that the *IR* on the final year equals the appropriate market rate of interest. This is discussed in more detail in the next chapter.

Divisible investment

Human capital investment does not always take place at school. Nor is human capital investment always an indivisible type of decision in which people devote themselves only to full-time investment. Often investment opportunities come in smaller units: one can go to school part time, one can take an adult education course, or one can train 'on the job' while simultaneously working. Many jobs, such as a management trainee or an accountant, offer wide training opportunities. Other occupations such as cab driver or waiter are more 'dead end' – they offer little incentive to train on the job as the job is simpler and it changes less quickly. Nevertheless individuals even in these jobs can opt to train, either on the job, or off it (at night school for example).

In the human capital model individuals are envisaged as spending some time and effort in improving themselves at every stage of their lives. The amount of time and effort will vary, being greater for youthful individuals, and for individuals just starting a job. The method of self-investment will also vary. Sometimes it will take the form of formal training courses, at other times it will consist of taking jobs which offer learning opportunities – 'on the job' training. The different types of training will be covered in later chapters. Here we are interested in laying down the broad outlines of the investment process, the basic imperatives, as it were, to which all people must respond.

How does one know whether to invest on a full-time or part-time basis? The problem is difficult but was first tackled by Yoram Ben-Porath in a classic article written in 1967.

2.4 The Ben-Porath model

Essentially Ben-Porath claims that in every year of one's life, one invests in oneself – buys eds – in accordance with the benefits and costs of buying the eds at that stage of the life cycle. Benefits are equal to the present value of the extra wages obtainable from the incremental unit of training. Costs are primarily the foregone earnings entailed in diverting one's time to acquiring that incremental unit. Ben-Porath assumes that individuals behave much like firms. Just as firms produce so that marginal cost equals marginal revenue, so individual investors purchase human capital up to the point that marginal cost equals (the present value of) marginal gain.

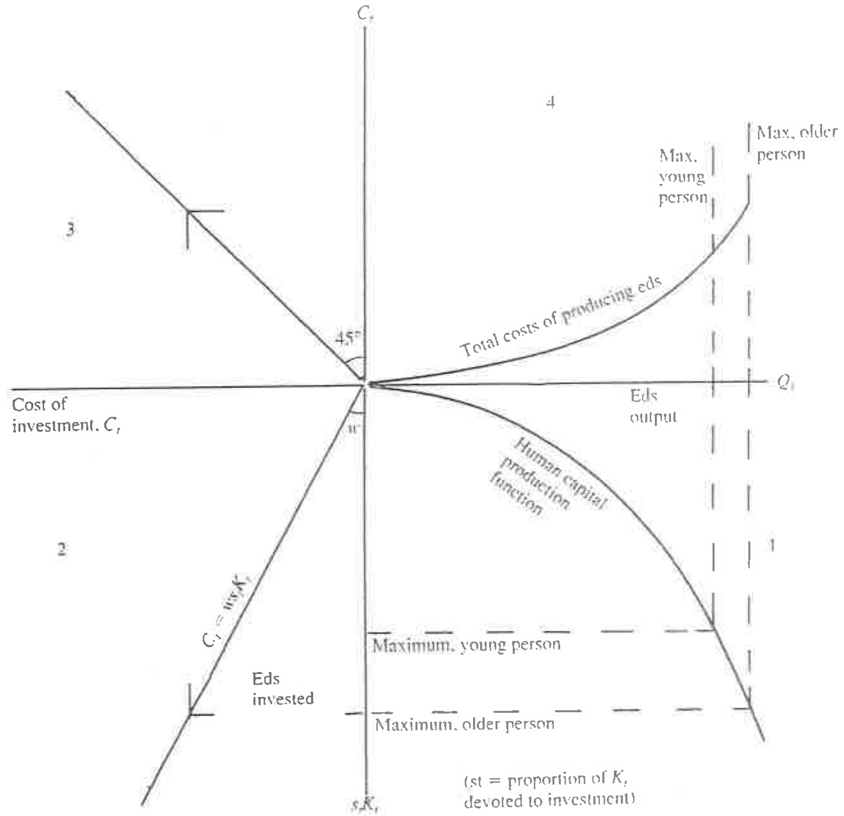

Figure 2.5 The human capital production function and the total cost of producing eds

Let us consider the costs side first. The elements of the model are laid out in figure 2.5. The central component is the 'human capital production function' given in quadrant 1. Eds are produced every year, Q_t, and the input is the human capital the person diverts from the market into self-investment. We measure this input as s_tK_t, where s_t is the proportion of human capital stock, K_t, diverted from earnings. In other words, the person can use his or her eds to earn money in the market, or use the eds to study and produce further eds. A simple human capital production function is:

$$Q_t = (s_tK_t)^b, \qquad (2.1)$$

where $0 < b < 1$ is an 'ability' parameter. The individual's decision problem is to choose s_t every year.

Note that since s_t cannot be greater than unity, the maximum Q that can be produced in any given year, t, is limited by the existing capital stock, K_t. However, since K_t grows with age (assuming investment continues), the maximum possible output of human capital, Q, grows with age. This growth in the maximum is indicated on the diagram, and illustrated in table 2.1. Table 2.1 shows an individual with an assumed initial stock of human capital of 100 'eds', *and an exponent, b, equal to 1/2. The first column shows possible human capital output in this initial year for different investment time fractions, s_t. Thus human capital increases by 7.07 units per period with investment time intensity $s_t = 50\%$, for example. It can increase at most by 10 units with maximum time intensity, $s_t = 1$. With maximum investment in the first year, the stock of eds would increase by 10. Then at the beginning of the next time period the individual would have 110 eds, and if he or she continued to invest full time the* maximum production of eds would be 10.49 units. Particularly at young ages, as we will see, this upper limit on the amount that can be invested will be important.

It might seem strange to have eds both as an input and as an output. An analogy would be the case of corn. Corn can be either eaten, or saved and used as seed for next year's crop: that is, corn can either be consumed or invested. In the same way we take it that eds can be consumed (by renting them in the market) or invested so as to produce further eds.

Investing the eds to produce further eds has a foregone earnings cost (and perhaps direct costs such as the purchase of books and training materials, which we ignore for simplicity). The foregone earnings cost, C_t, is the wage per ed, w, multiplied by the number of eds directed away from the market and towards investment:

$$C_t = ws_tK_t. \tag{2.2}$$

This function is graphed in quadrant 2.

Using the 45 degree line of quadrant 3 to map dollar costs over to quadrant 4, we then have in quadrant 4 what we want: the total cost function for producing eds. Notice how, when the maximum Q_t is reached, the cost function ends (dotted vertical line). This maximum will move outward with age, as already explained.

The formulation of the human capital production function emphasises the individual's own resources. Borrowing funds for self-investment is assumed to be unimportant – as is realistic, since human capital is not good collateral for a bank loan (see next chapter). More restrictively, there might appear to be no role for family funds to enter the investment decision model – yet such funds are likely to be vital in the early years, in that wealthy families can afford to wait longer for their children to become self-supporting. However family wealth can be shown to have a role in the model in that a wealthy family will

Table 2.1. *Illustrative human capital production function*

s_t	$K_0 = 100$ Q_0	$K_1 = 110$ Q_1	$K_2 = 120.5$ Q_2	$K_3 = 131.5$ Q_3
0	0	0	0	0
0.1	3.16	3.32	3.47	3.63
0.2	4.47	4.69	4.91	5.13
0.3	5.48	5.74	6.01	6.28
0.4	6.32	6.63	6.24	7.25
0.5	7.07	7.42	7.76	8.11
0.6	7.75	8.12	8.50	8.88
0.7	0.37	8.77	9.18	9.59
0.8	8.94	9.38	9.82	10.26
0.9	9.19	9.25	10.11	10.88
1.0	10.00	10.49	10.98	11.47

Notes: Human capital production function assumed:

$$Q_t = (s_tK_t)^b, \text{ where } b = 1/2$$

Q_t = output of capital
s_t = fraction of human capital stock (K) devoted to production of further capital = portion of time devoted to study in a year.
b = 'ability' parameter, $0 < b < 1$ (if $b = 1$ there are no diminishing returns).

apply a lower discount rate when evaluating the benefits of investment. This will lead to more investment, as is now shown.

On the benefits side, the benefit of an extra unit of human capital is the present value of the stream of future wages, PV(w, i), which that unit will bring. The stream goes on until retirement, age sixty-five, say. Thus benefits, B_t, are:

$$B_t = \text{PV}(w, i)\, Q_t \tag{2.3}$$

$$= \frac{w}{i}\left(1 - \frac{1}{(1+i)^{65-t}}\right) Q_t$$

$$\cong \frac{w}{i} Q_t \text{ when a person is young } (t \text{ small})$$

$$\cong 0 \quad \text{when a person nears retirement } (t \cong 65).$$

The above is a simplification because future benefits are not known with certainty (see Levhari and Weiss, 1974 and Warren and Snow, 1990). Also, the retirement age is itself chosen (see chapter 5). However it is a useful starting point. The benefits schedule is shown in the top panel of figure 2.6, which also includes the total cost function already derived. The benefits schedule is a straight line from the origin. The slope is steeper for young than old workers because the present value of a unit of human capital is greater for younger workers. The slope will also be steeper if the discount rate, i, is low – that is, if the person can afford to wait. This position, as mentioned above, is more likely to characterise individuals from wealthy than poor families.

Where the difference between total benefits and costs is greatest gives the surplus maximising output of human capital. As drawn, this is output Q_1 for the young person. At Q_1 the slopes of the benefit and cost curves are equal, that is, marginal benefit equals marginal cost. This is shown in a different way in the bottom panel, which graphs the marginal benefit and marginal cost curves.

Because the present value of a unit of human capital is greater for young than old workers, the optimum output of human capital falls over the life cycle. As the benefit curve in the upper panel of figure 2.6 falls with age, so the marginal benefit curve in the lower panel declines. Assuming the marginal cost curve remains unchanged with age, the MB curve simply slides down the MC curve. Two positions are shown in figure 2.6, with Q_1 for the younger worker, and Q_2 for the older.

Not all individuals have the same marginal cost and marginal benefit curves. Presumably those individuals with greater abilities would find it cheaper to 'produce' human capital. For example, clever people might find that it takes less time to learn calculus in school or management techniques on the job. For them the marginal cost curve would be lower, and greater amounts of human capital would be purchased.

Let us now follow a typical man through his life (women's intermittent labourforce participation makes their earnings profile more complicated – this is analysed in chapters 5 and 6). Assume for simplicity that the marginal cost curve does not vary as an individual gets older – except that the vertical section indicating maximum Q_t shifts out with age. During the initial years, the present value of gain for each unit of investment is relatively high, and maximum Q_t low – so the individual will be at a corner, producing as much Q_t as possible, that is, choosing a time investment fraction, s_t, of unity (no time spent earning). Then, as the individual ages, the maximum shifts outwards, and the marginal benefit curve shifts downwards. It shifts continuously until the year in which he retires, and in that year the present value of the marginal gain curve coincides with the horizontal axis. In that year, the last year of his work life, investments in market earnings power have no value (from an investment viewpoint – though they might still be undertaken for reasons of enjoyment, i.e., consumption reasons) since he plans to cease working.

Figure 2.7 translates these investment patterns into a life-cycle investment curve and corresponding human capital stock curve. The figure also shows the time path of s_t, the fraction of time spent investing every year. According to our argument, the amount of investment in eds, Q_t, increases for a period since

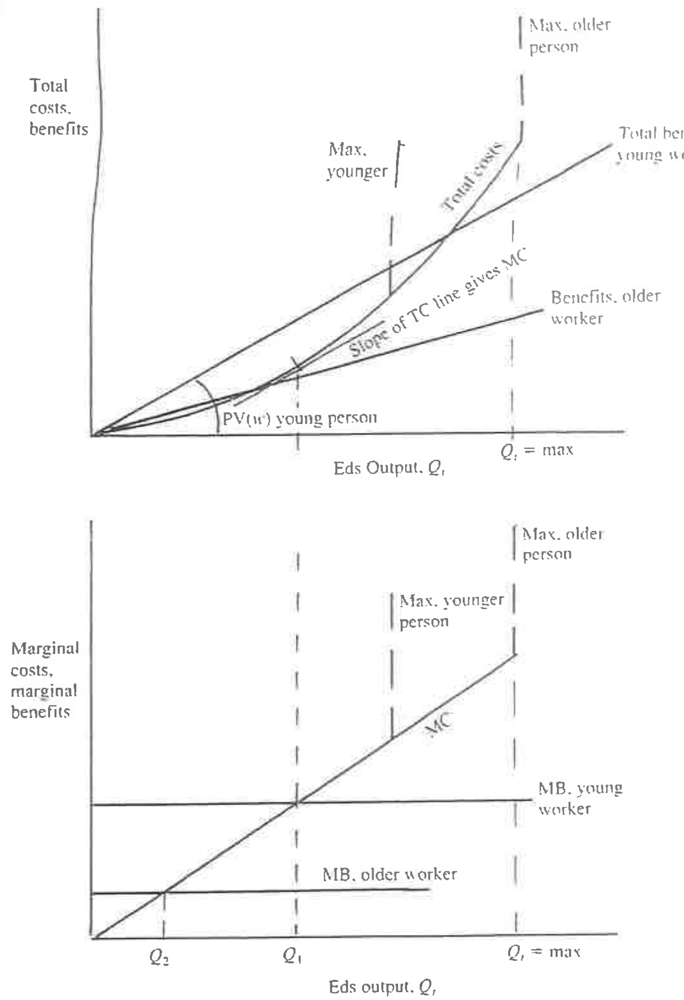

Figure 2.6 Total and marginal benefits and costs of producing eds

initially human capital is a very profitable investment, but the individual cannot produce enough of it, and so chooses $s_t = 1$. Q_t then declines continuously as one gets older reaching zero at retirement. Accumulated human capital is computed by adding the annual investments. The process of adding yearly human capital investments yields a stock of human capital curve, K_t, as depicted in figure 2.7. Note that the stock of human capital increases quickly in the period when $s_t = 1$, then more slowly in middle age, and stops increasing at all when one is old. In fact, ignoring depreciation (see below) human

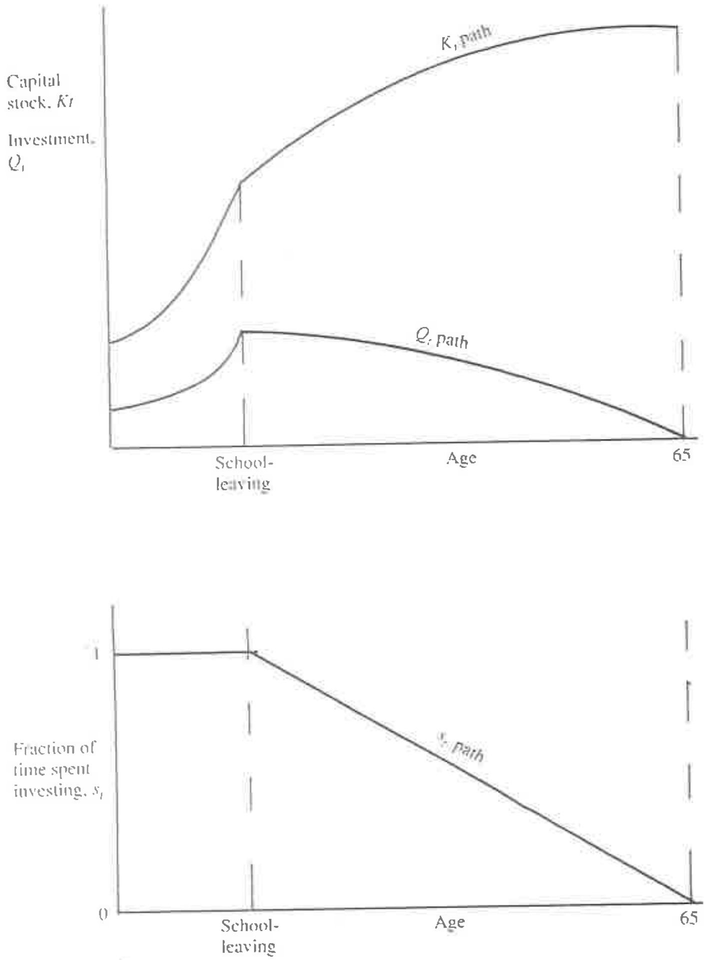

Figure 2.7 The production of human capital and the stock of human capital over the life cycle

capital stock peaks at retirement and reflects a concave function looking very much like the age–earnings profiles depicted earlier in the chapter.

This should be no surprise: earnings in the market place are determined on the basis of one's accumulated human capital stock. Human capital stock and earnings are related in the following way. Define potential earnings, E_t, as the most that an individual aged t could earn if he spent all his time working. Earnings in this case would be equal to the product of the stock of human capital accumulated over past investments, K_t, and the wage rate per unit of

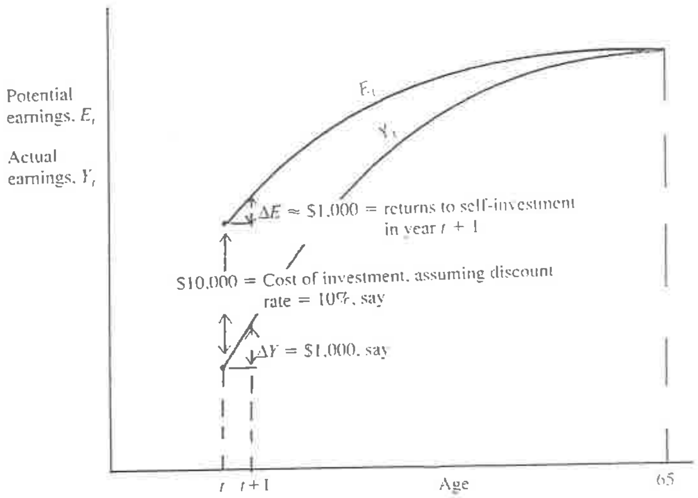

Figure 2.8 The relation between actual (Y) and potential (E) earnings

human capital, w. w is given by the market, and is assumed constant over life and independent of the stock of human capital. Thus:

$$E_t = wK_t. \tag{2.4}$$

Then observed earnings, Y_t, equals potential earnings minus the foregone earnings cost, C_t $(= ws_tK_t)$, of human capital investment in that year:

$$\begin{aligned} Y_t &= E_t - C_t \\ &= wK_t(1 - s_t). \end{aligned} \tag{2.5}$$

For example, if we measure eds so that the wage per ed is \$1 per ed per year (say), then potential earnings follow a path the same as the capital stock path, and actual earnings are related to potential earnings by the fraction $(1 - s_t)$. As illustrated in figure 2.8, observed earnings are much smaller than potential earnings (and the value of the individual's capital stock) early in life, when s_t is large, and the gap tapers to nothing as s_t declines.

It is instructive also to link potential and actual earnings using the internal rate of return. When investment costs, C_t, are incurred, the value of human capital and thus of potential earnings is increased by some amount ΔE_t. The relationship between the two is given by the internal rate of return, IR, which should equal some market rate of interest, i, in equilibrium:

$$\Delta E_t / C_t = IR \cong i.$$

For example if \$1,000 of earnings is foregone by going on a training course, so

C_t = \$1,000, and capacity earnings are increased by \$200 a year as a result of the course, so ΔE_t = \$200, then $IR = 20\%$. This is a very good rate of return, so more investment should be undertaken until IR comes down to about $i = 10\%$, say. Once this happens, given that $Y_t = E_t - C_t$, we have:

$$Y_t = E_t - \Delta E_t/i,$$

and:

$$E_t - Y_t = \Delta E_t/i.$$

However, $\Delta E_t \equiv \Delta Y_t$, as figure 2.8 shows. Therefore the gap between potential and actual earnings can be written:

$$E_t - Y_t = C_t = \Delta Y_t/i. \quad (2.6)$$

This is an important equation, because it shows that we can simply build up a person's *potential* earnings profile using that person's *actual* earnings profile. The equation shows that the increase in observed earnings in a given year, divided by a suitable discount rate ('capitalised') allows us to assess the amount that person invested in himself or herself in that year, and therefore his/her hypothetical potential earnings. Suppose, for example, that a man was earning \$10,000 at the beginning of the year, his pay increased by \$1,000, and the discount rate is 10%. Then, from equation (2.6), his self-investment in new skills during that year must have been approximately \$10,000 (= 1,000/0.1), and his potential earnings \$20,000.

2.5 Algebraic depiction

It is useful to collect together the equations we have used above, and derive some algebraic results.

We have already (equation 2.4) defined potential earnings, E_t, as the amount an individual aged t could earn by spending all his time working:

$$E_t = wK_t.$$

The path of E_t is therefore determined by the path of K_t, which is in turn determined by how much one invests during the year, that is, the path of Q_t in figure 2.7. This is seen algebraically: from the definition of E_t it follows that:

$$\partial E_t/\partial t = w\partial K_t/\partial t = wQ_t.$$

To explain movements in E_t we must thus analyse movements in investment, Q_t.

We can depict the process of human capital creation by means of a production function such as that used in figure 2.5. For illustration we use a simplified version of the Cobb–Douglas production function as suggested in equation (2.1) above. We assume capital created in any year, Q_t (which equals the rate of change of capital stock from year to year) is related to the fraction of time, s_t, devoted to enhancing one's existing skill level, K_t. Investment effort, s_t, is measured as a proportion of total time available and hence can vary from zero to one. Thus:

$$Q_t = \Delta K_t/\Delta t = (s_tK_t)^b,$$

where b is a constant less than unity depicting 'ability'.

From the production function, it is possible to derive cost curves for the production of human capital. Since the cost of producing human capital is the foregone earnings cost of taking time away from work to produce the human capital, total costs can be computed as opportunity costs. Thus as noted in equation (2.2) above, total dollar costs of producing human capital at age t, C_t, are:

$$C_t = ws_tK_t.$$

This cost function can be expressed in units of human capital Q_t. From the production function we have:

$$s_tK_t = Q_t^{1/b},$$

so:

$$C_t = wQ_t^{1/b}.$$

This implies a marginal cost function:

$$\partial C_t/\partial Q_t = \frac{w}{b}Q_t^{(1-b)/b},$$

which, it must be remembered, becomes vertical when $s_t = 1$ (see figure 2.6).

The benefits curve is (equation (2.3) above):

$$B_t = \frac{w}{i}\left(1 - \frac{1}{(1+i)^{65-t}}\right)Q_t,$$

and the marginal benefits curve is:

$$\partial B_t/\partial Q_t = \frac{w}{i}\left(1 - \frac{1}{(1+i)^{65-t}}\right).$$

At any age, optimal investment is determined by equating marginal cost and marginal benefits from investment. Thus setting $\partial C_t/\partial Q_t$ equal to $\partial B_t/\partial Q_t$ yields:

$$\frac{w}{b}Q_t^{(1-b)/b} = \frac{w}{i}\left(1 - \frac{1}{(1+i)^{65-t}}\right).$$

Solving for Q_t gives:

$$Q_t = \left(\frac{b}{i}\left(1 - \frac{1}{(1+i)^{65-t}}\right)\right)^{b/(1-b)}. \quad (2.7)$$

Equation (2.7), shows us that human capital output, Q_t (and thus earnings) depends upon i, b, and t. Looking first at the discount rate, i, we see that Q_t is smaller the higher is i. In other words individuals facing high discount rates will invest less, accumulate less capital, and consequently have a lower growth in earnings over their lives. In the next chapter we will see that poor families face higher discount rates, and hence can be expected (on the basis of the human capital model) to accumulate less capital. This is the equity basis for subsidising education.

Also according to equation (2.7), Q_t rises with b, the indicator of 'ability'. The more able accumulate more human capital. This seems plausible – a higher b lowers the marginal cost curve in figure 2.6, and causes more Q_t for a given MB curve. However, the more able need not remain at school (full time investment) longer than the less able. Equation (2.6) only holds when MC can be equated with MB. But during the schooling period, as we have seen, MB is greater than MC because the MC curve reaches a maximum. This maximum shifts out with age. It is possible that the maximum shifts outward more quickly for the more able people, and so their period of schooling (though not their output of Q_t) is in fact shorter than average.

Finally, whatever the value of b and i, Q_t decreases as t increases. Eventually Q_t falls to zero at the age of retirement, when $t = 65$. In sum, the capital stock – and potential earnings – grows more quickly with high b, less quickly with high i, and, after an initial period of swift increase, increases less quickly with age.

To derive actual pay, Y_t, subtract investment costs, C_t, from potential earnings, wK_t. Thus:

$$Y_t = wK_t - ws_tK_t = w(1 - s_t)K_t.$$

Since K_t is increasing and s_t decreasing over the life cycle, observed earnings must increase, and the rate of increase of observed earnings must exceed the growth in capacity earnings (see figure 2.8).

2.6 Depreciation

The earnings profile of figure 2.8 peaks when one is about to retire, yet the earnings profile of figure 2.1 peaks slightly before retirement age and then is negatively sloped. Why the inconsistency?

Skills depreciate and often become obsolete. What this means is that one's stock of human capital can deteriorate with age. How often do we wish that we remembered everything we learned in our freshman courses? It is the forgetting of skills, the obsolescence of skills, and perhaps the decline in health which cause depreciation. Few estimates of depreciation exist. Thus one would be hard pressed to postulate just how depreciation varies over the life cycle. Nevertheless, it is intuitively plausible to hypothesise that depreciation of skills probably increases with age. Depreciation can be illustrated graphically (a mathematical analysis is given in the appendix to chapter 4).

Figure 2.9 depicts depreciation as well as the entire investment/earnings process. On the top half is investment and on the bottom are earnings profiles. Curve C_t is the (gross) investment path, and depends on Q_t. The upward sloping curve, $D_t = \delta PK_t$, represents depreciation for each year, where δ is the depreciation percentage and PK_t is the dollar value of human capital. For example, given that w = \$1 per year is the rental rate of an ed, $P = w/i$ is the capitalised value of an ed, and P can be thought of as the price of human capital. Thus, if i = 10%, then P = \$10. If the number of eds is $K = 20$, then the earnings stream is \$20 (= $iPK = wK$), and the dollar value of eds is \$200 (= PK). If δ = 3% a year then $D = \delta PK$ = \$6.

The difference between investment and depreciation is net investment, C_{nt}. Thus $C_{nt} = C_t - \delta PK_t$. Net investment equals zero when depreciation equals gross investment. Net investment is negative when one's capital stock depreciates more quickly than it appreciates. This occurs when depreciation exceeds gross investment (see the appendix to chapter 4).

Potential earnings, when depreciation is zero, rise continuously as depicted in figure 2.8. When depreciation is non-zero, potential earnings peak when net investment is zero. Beyond this point capital stock is declining as depreciation exceeds investment. Thus earnings decline from this point on. As in figure 2.9 observed earnings can be obtained by subtracting investment costs from E_t. Again, as investment costs go to zero, both earnings streams converge.

2.7 Conclusions

Observed earnings profiles are concave. They rise initially at a rapid rate, then at a decreasing rate, finally peaking and then falling. The chapter develops a life-cycle human capital investment model to explain these patterns. Earnings are related to skills and skills are acquired through human capital investment both in school and 'on the job'. As skills accumulate over life, earnings rise in direct proportion to accumulated human capital. The finite life constraint governs incentives for investment. Older workers about to end their work life by entering retirement have few years in which to reap investment gains. Young workers with a whole work life ahead have many years in which to reap

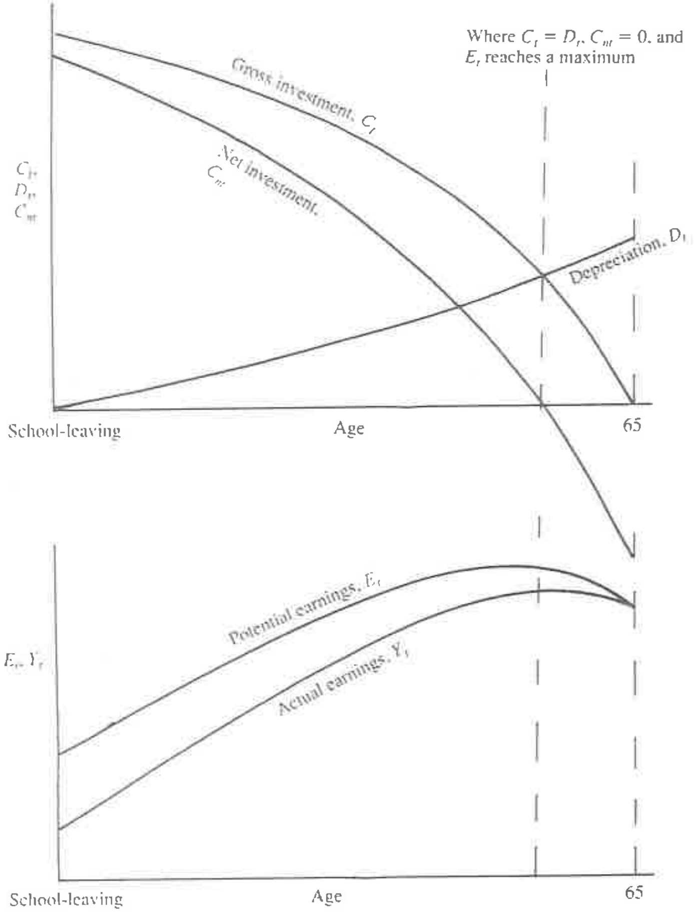

Figure 2.9 The investment–earnings process

gains. Therefore the young invest more heavily, thereby accumulating human capital at a more rapid rate – initially at an increasing rate. Over the life cycle total capital stock is theorised to rise at a diminishing rate, peaking some years before retirement, then depreciating.

Potential earnings are proportional to capital and thus mirror the time profile of accumulated human capital assets. Observed earnings differ from potential earnings by the amount the individual is investing, and considerably understate potential earnings early in the life cycle. Concavity of the earnings profile has thus been explained. We leave for the next chapter the question of why education raises the height of earnings profiles.

Appendix 2.1 Present values and discounting

The simplified annuity formula

Suppose an investment is expected to bring in \$1,000 a year over a certain period, say four years. By using the formula for calculating present values we can work out what this investment would be worth now as a capital sum. To calculate present value requires 'discounting' the future stream of returns. The rate of discount usually used is the market interest rate since this shows the opportunity cost of funds. The estimated present value of the investment is then compared with its asking price to see if it is worth making.

Data for an illustrative calculation are given in table A2.1. This applies the present value formula using two discount rates, 5% and 10%, to an investment providing \$1,000 per year income for four years. The present value formula is:

$$\mathrm{PV}(i) = \frac{Y_1}{(1+i)} + \frac{Y_2}{(1+i)^2} + \frac{Y_3}{(1+i)^3} + \ldots + \frac{Y_t}{(1+i)^t},$$

where PV(i) is the present value of the income stream discounted at interest rate i measured in decimal points, i.e., 5% is written as 0.05; $Y_1, Y_2, \ldots$, etc. is the income received each year; t is the number of years over which income is expected to be received.

In the example of table A2.1, $t = 4$, and $Y_1 = Y_2 = Y_3 = Y_4 = \$1{,}000$. The present value of the investment can be seen to be \$3,546 if $i = 0.05$, and \$3,169 if $i = 0.10$.

The present value formula above can be written more simply where the income (Y) received is the same each year as in our example. The formula is then called the 'annuity' formula:

$$\mathrm{PV}(i) = (Y/i)(1 - 1/(1+i)^t).$$

This is derived as follows. Using the basic formula and assuming $Y_1 = Y_2 = Y_3 = \ldots = Y_t = Y$, we can write

$$\mathrm{PV}(i) = Y[1/(1+i) + 1/(1+i)^2 + \ldots + 1/(1+i)^t].$$

The expression in square brackets is a 'geometric progression' (GP) with each succeeding term multiplied by a common factor, $1/(1+i)$. The formula for taking the sum of a GP is:

$$A(1 - F^t)/(1 - F),$$

where A is the first term ($1/(1+i)$ in our case), t is the number of terms, and F is the common factor (also $1/(1+i)$ in our case). Substituting into the formula gives:

Table A2.1. *Present value of $1,000 per annum received for 4 years, at various discount rates*

Return	$1/(1+0.05)^t$	5%	$1/(1+0.10)^t$	10%
$1,000 at end of first year, $t = 1$	0.952	$952	0.909	$909
$1,000, $t = 2$	0.904	$907	0.826	$826
$1,000, $t = 3$	0.864	$864	0.751	$751
$1,000, $t = 4$	0.822	$822	0.783	$783
Present value		$3,546		$3,169[a]

Note:
[a] Notice how $3,169 can also be derived from the annuity formula, $PV(i) = (Y/i)(1 - 1/(1+i)^t)$ using $i = 10\%$ and $t = 4$. Thus: $PV(10\%) = (\$1{,}000/0.01)(1 - 1.1^{-4}) = \$10{,}000\,(1 - 0.68) = \$3{,}169$.

$$PV(i) = (Y/i)(1 - 1/(1+i)^t), \text{ as above.}$$

This formula becomes simpler still if t is large, say, above 30, because then the last term in the brackets becomes approximately zero. For example if $i = 10\%$ and $t = 30$ then $1/(1+i)^t = 0.054$; and if $t = 50$, $1/(1+i)^t = 0.0085$. So if t is greater than thirty years we can write:

$$PV(i) \cong Y/i.$$

Thus $1,000 received for thirty years and discounted at 10% would have a present value of approximately $10,000. This is the *simplified annuity formula* and is used frequently in human capital analysis.

Notice how the present value of an investment is higher: the higher are the annual returns from the investment (Y), the lower is the discount rate (i), and the larger is the period over which the returns are forthcoming (t). This is what you would expect using common sense. The question we now face is what discount rate should be used to compute the PV: To answer this we have to bring the asking price of the investment into the analysis, and also consider what alternative investments are available.

Investment appraisal

Continuing with the example of table A2.1, suppose that the asking price (C) of the investment were $3,546. We know that PV(5%) = $3,546, so the problem is whether to buy the investment. Whether we buy the investment or not depends on whether we can get more than 5% on our money elsewhere.

The rate of discount usually used to evaluate investments (the criterion rate

of discount) is the market rate of interest. For example suppose that 10% was being offered for funds deposited at the local bank. Then $3,169 deposited at the bank could provide $1,000 per annum for four years – with nothing in the account at the end of the period. The reasoning is as follows: deposit $3,169 at the beginning of year 1. By the end of the year it has grown to $3,169 + $317 = $3,486. Pay out $1,000. The beginning balance in year 2 is then $2,486 which grows to $2,486 + $249 = $2,735. Pay out $1,000. The beginning balance in year 3 is then $1,735 and the process continues. At the end of year 4 exactly $1,000 remains to be paid out.

This $1,000 per year for four years at an initial cost of $3,169 is better than an investment which costs $3,546 for the same income stream. In fact only if the price of the investment fell to $3,169 would it even be worth contemplating. And since human (and industrial) investments are risky, while the local bank is not, the price would have to fall considerably below $3,169 before it were worth buying the investment given market interest rates of 10%.

Before going any further it is necessary to define the 'internal rate of return' (IR) on an investment. The IR is a special rate of discount:

$$PV(IR) = C.$$

In other words the internal rate of return ('rate of return' for short) is that rate of discount which makes the present value of income from an investment equal the investment's asking price. For example in table A2.1 the IR for the investment if $C = \$3{,}546$ is 5%. If C dropped to $3,169 the IR would rise to 10%.

When working out whether to buy an investment or not, that is, when 'appraising' an investment, it has to be determined whether the investment's IR is greater than the market interest rate (r) plus an appropriate risk premium (p). In symbols the rule is:

if $IR > r + p$, buy the investment;
if $IR < r + p$, do not buy.

We know that $C = PV(IR) = Y/IR$ so that $IR = Y/C$ using the simplified annuity formula. Therefore it follows from the above rules that, if an investment is to be bought:

$$Y/C > r + p.$$

For example if the market interest rate were 10%, and the risk premium 5%, Y/C would have to be greater than 15%.

The investment rule can be translated into just a comparison of the asking price and the present value of returns. The present value of the returns evaluated at a discount rate of $r + p$ is:

$$PV(r + p) = Y/(r + p).$$

If $Y/C > r + p$ in accordance with our previous rule, then $Y/(r + p) > C$ and therefore $PV(r + p) > C$. Thus if the present value of the returns discounted at the market interest rate plus a risk premium is greater than the case price, the investment should be considered.

Why discount the future?

Two answers to this question are generally given. The first is that individuals as consumers prefer present satisfactions to future satisfactions. The average person has a 'positive time preference' in the jargon, or, to put it in another way, he or she discounts the future at some rate. In order to persuade people to save and lend their money to business a positive rate of interest has therefore to be given. Thus for $100 given up now, $110 has to be promised in the future if people discount the future at 10% per annum.

The other answer to the question relates to the productivity of investment. By investing in machinery we can make $100 invested now grow into $110 next year (or more or less depending on the quality of the investment). It is because more can be produced with machinery than without, that businessmen can in fact pay potential savers a sufficient rate of interest to make them overcome their time preference and save.

Notice that in all the above we have been ignoring inflation, that is, taking real and money rates of interest as equal. In practice rates of interest are quoted in money or 'nominal' terms. The relation between the real and nominal rate of interest is approximately:

$$r = M - \Delta P/P,$$

where r is the real rate of interest, M is the money rate of interest, $\Delta P/P$ is the rate of inflation. The long-term real rate of interest is probably 3% to 4% (this was the rate assumed by economists in the eighteenth and nineteenth centuries, before inflation). If inflation is about 10%, then the nominal rate of interest is 13% to 14%. Given a nominal return on your investment you subtract the rate of inflation to get the real return.

3 Schooling

3.1 Introduction

In chapter 2 earnings profiles were introduced. It was argued that these profiles were related to human capital stocks accumulated over the life cycle. Human capital stocks were determined by individuals rationally choosing the proportion of time devoted to investment at each age. A rationale based on the finite life constraint served as reason for greater specialisation in investment activity for the young compared to the old. This led to greater human capital accumulation at young ages and hence a concave earnings profile.

In this chapter we concentrate on the initial life-cycle phase. Here specialisation occurs so that individuals literally spend all their time devoted to human capital investment. Such specialisation is defined as schooling. The questions we ask are: How does one picture potential earnings during the schooling phase? Can the returns to schooling be analytically measured? Why do individuals differ in their amounts of school? And finally, what role does government policy play in influencing both the levels of schooling across the population and the returns to the educational process?

3.2 Human capital specialisation

It is useful to define 'schooling' as being the period in which 100% of one's time is devoted to earnings enhancement. Recall the human capital production function described in chapter 2 and illustrated in table 2.1. The production function shows the amount of human capital created as a function of one's time investment fraction, s_t, holding constant innate ability and initial capital stock. Because the time investment fraction can at most be 100%, human capital output, Q_t, has an upper bound. This upper bound is depicted in figure 3.1. As drawn, the marginal benefits of investment are so high that we obtain full-equilibrium on the vertical region of the marginal cost curve, implying full-time investment. This must be true during all years in which the individual is going to school.